1.

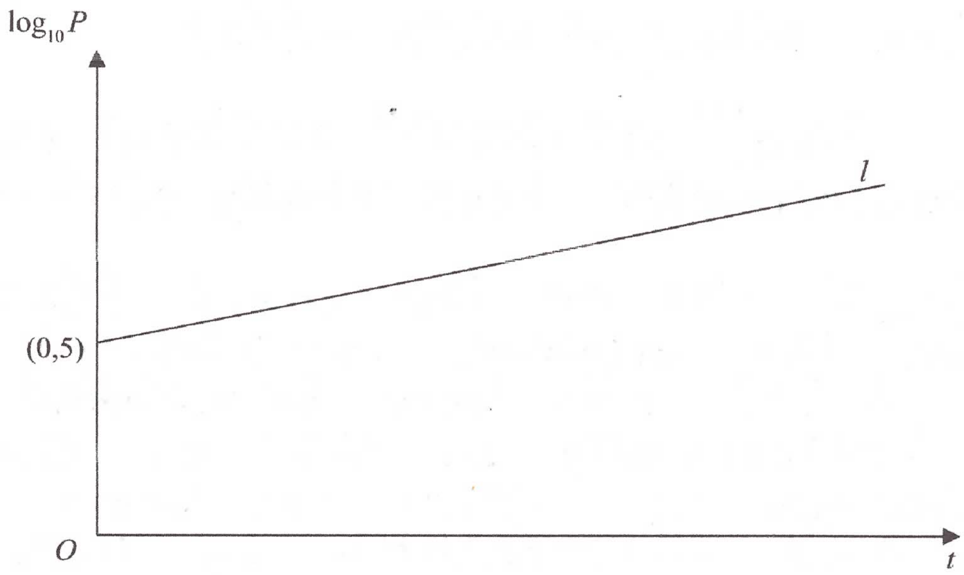

Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10} P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,

(ii) the value of the constant b.

(2)

(d) Find

(i) the population predicted by the model when $t = 100$, giving your answer to the nearest hundred thousand,

(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

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Question continued

(a) $y = mx + c \Rightarrow \log_{10} P = mt + c$

$$\boxed{\log_{10} P = \frac{1}{200}t + 5}$$

(b) As $P = ab^t$, then $\log_{10} P = \log_{10}(ab^t)$

$$\log_{10} P = \log_{10} a + \log_{10} b^t$$

$$\log_{10} P = \log_{10} a + t\log_{10} b$$

$$\log_{10} a = 5 \quad \text{and} \quad \log_{10} b = \frac{1}{200}$$

$$a = 10^5, \quad b = 10^{(1/200)}$$

$$\boxed{a = 100{,}000 \quad \text{and} \quad b = 1.0116 \ (4\text{s.f.})}$$

(c) (i) a is the <u>initial population</u>

(ii) b is the <u>proportional increase of population each year</u>

(d) (i) $P = (100{,}000)(1.0116^{100})$

$= 316{,}227.766 = \underline{300{,}000}$ (nearest hundred thousand)

(ii) $200{,}000 = (100{,}000)(1.0116^t)$

$$2 = 1.0116^t$$

$t = \log_{(10^{\frac{1}{200}})}(2) \Rightarrow \underline{t = 60.2 \text{ years}}$ (to 3s.f.)

(e) • The model predicts that growth never stops.

• 100 years is too far away to predict populations.

(Total for Question is 13 marks)

2.

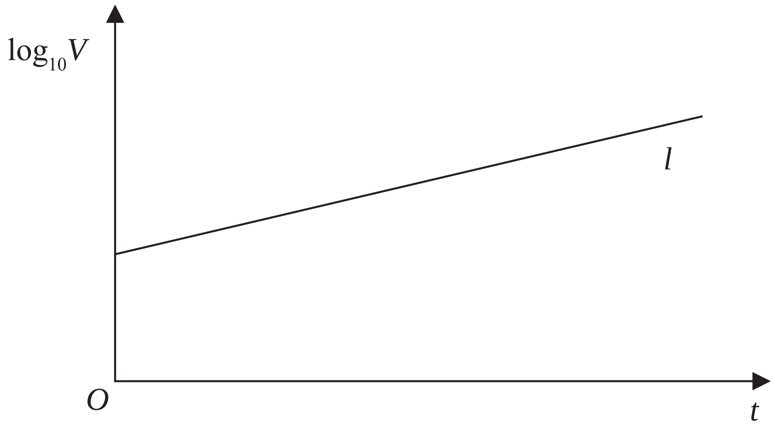

Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10}V$ since 1st January 1980.

The equation of line l is $\log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,

(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

a) $p = 10^{4.8}$ $\quad$ $q = 10^{0.05}$

$= 63095.7$ $\quad$ $= 1.122018$

$\simeq 63100$ $\quad$ $\simeq 1.122$

bi) value of painting on 1st January 1980

ii) The proportional increase of the value each year

c) $2010 - 1980 = 30$

$\log_{10} V = 0.05(30) + 4.8$

$V = 10^{6.3}$

$= 1995262$

$\simeq £2000000$

3. The value of a car, £V, can be modelled by the equation

$$V = 15\,700\mathrm{e}^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925\mathrm{e}^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

a) $\underline{t=0}$: $V = 15700e^{0} + 2300$

$= \boxed{£18000}$

bi) at $t = T$, $\frac{dV}{dt} = -500$

$\frac{dv}{dt} = (-0.25)15700e^{-0.25t}$

$= -3925e^{-0.25t}$ //

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Question continued

at $t = T$: $-3925e^{-0.25T} = -500$

$(\times -1)$

$\Rightarrow 3925e^{-0.25T} = 500$ //

ii) $e^{-0.25T} = \frac{500}{3925} = \frac{20}{157}$

$\therefore \ln\left[e^{-0.25T}\right] = \ln\left[\frac{20}{157}\right]$

$-0.25T = \ln\left[\frac{20}{157}\right]$

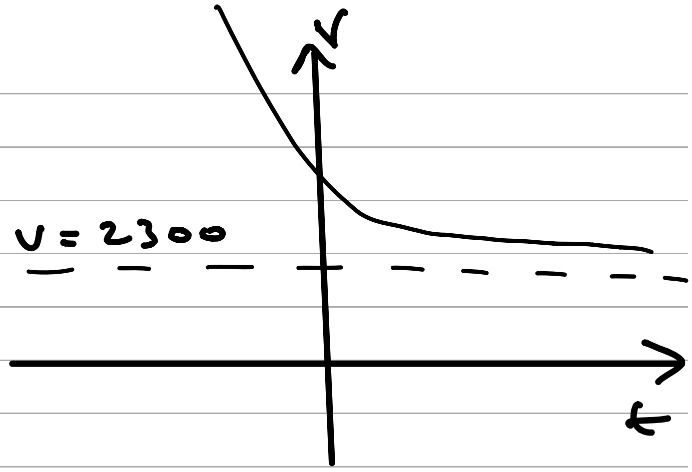
$T = -4\ln\left[\frac{20}{157}\right]$

$T = 8.24$ years //

$\boxed{\text{8 years, 3 months}}$

Question continued

c) $\boxed{£2300}$



d) For large values of t the car is likely to be worth less than £2300, eventually.

4. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.

(1)

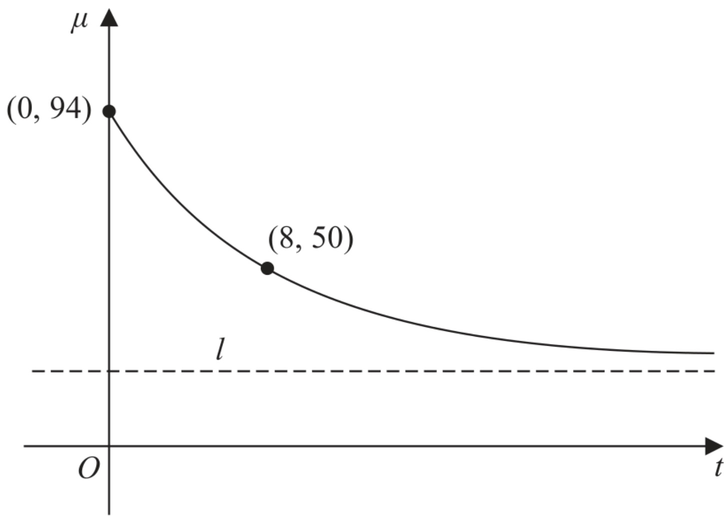


Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

a) placed on table @ $t = 0$: $\theta = 18 + 65e^{0}$

temperature $= 83$°C

Question continued

b) $\theta = 35 = 18 + 65e^{-\frac{t}{8}}$

$\Rightarrow 65e^{-\frac{t}{8}} = 17$

$\Rightarrow \quad e^{-\frac{t}{8}} = \frac{17}{65}$

take ln of both sides : $-\frac{t}{8} = \ln\left(\frac{17}{65}\right)$

$t = -8\ln\left(\frac{17}{65}\right)$

$= 10.729\ldots$

$= 10.7$ (1 d.p.)

c) as $t \to \infty$, $e^{-\frac{t}{8}} \to 0$ from above so $\theta \to 18^{\circ C}$ from above.

hence, the minimum temperature ($18^{\circ C}$) is $> 15^{\circ C}$

d) $\mu = A + Be^{-\frac{t}{8}}$

given points $(0, 94)$ & $(8, 50)$: $94 = A + B^{0} \Rightarrow A + B = 94$ ①

$50 = A + B^{-\frac{8}{8}} \Rightarrow A + Be^{-1} = 50$

① − ②: $94 - 50 = B(1 - e^{-1})$

$44 = B(1 - e^{-1})$

$44e = B(e - 1)$

$B = \frac{44e}{(e-1)}$

Question continued

$A+B=94 \Rightarrow A=94-\frac{44e}{(e-1)}$

$=\frac{94e-44e-94}{e-1}$

$=\frac{50e-94}{e-1}$

as $t\to\infty$, $Be^{-\frac{t}{8}}\to 0$ so asymptote given by

$A=\frac{50e-94}{e-1}$ $(=\ell)$

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5. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live. Give your answer to 2 significant figures.

(2)

a) $\log_{10} V = 0.072t + 2.379$

raise both sides: $V = 10^{0.072t + 2.379}$ (base = 10)

$= 10^{0.072t} \times 10^{2.379}$

$\therefore a = 10^{2.379}$ & $b = 10^{0.072}$

by calculator, nearest whole value: $a = 239$, $b = 1.18$ (3s.f.)

$\Rightarrow V = 239 \times 1.18^t$

b) we get $V = ab$ when $t = 1$: $V = ab^1$. thus, the value of ab is the total number of views of the ad. 1 day after it went live

c) $t = 20$: $V = 239 \times 1.18^{20}$

$= 6545\ldots$

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Question continued

$\Rightarrow V = 6500$ views

6. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

(a) Let $t = 0$

$A = 80 - 45e^{ct}$

$= 80 - 45e^{0}$

$= 80 - 45$

$= 35 \text{ km}^2$ ※ 1

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Question continued

(b) From 2005 to 2019 = 14 years

Let $A = 60$, $t = 14$

$A = 80 - 45e^{ct}$

$60 = 80 - 45e^{c(14)}$

$60 = 80 - 45e^{14c}$ (1)

$45e^{14c} = 20$ (1)

$e^{14c} = \frac{20}{45}$

$= \frac{4}{9}$

$14c = \ln\left(\frac{4}{9}\right)$

$c = \frac{\ln\left(\frac{4}{9}\right)}{14}$ (1)

$c = -0.0579...$

$\therefore A = 80 - 45e^{-0.0579t}$ (1)

(c) The maximum area covered by trees is only 80km^2. (1)

Alternatively, you can substitute 100 into the equation :

$100 = 80 - 45e^{-0.0579t}$

$20 = -45e^{-0.0579t}$

$-\frac{20}{45} = e^{-0.0579t}$

$\therefore$ You can't solve this equation because you cannot take a log of a negative number

(Total for Question 11 is 6 marks)

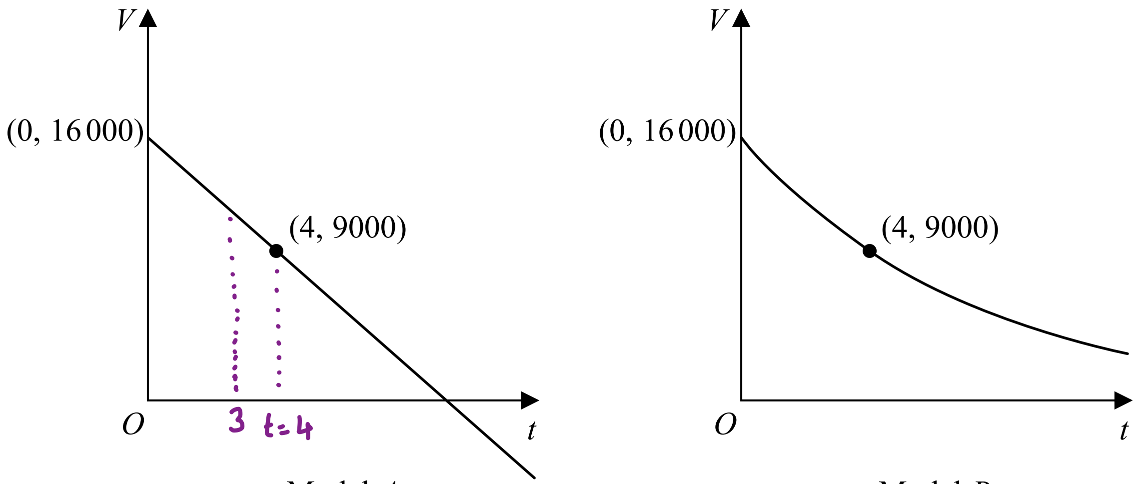
7. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

a i) $m = \dfrac{9000-16000}{4-0} = -1750 \Rightarrow y = 16000 - 1750x$

$V = 16000 - 1750t$

$V = 16000 - 1750 \times 3 = 10,750$ barrels ①

a ii) • $V = 16000 - 1750t$ What happens when $t = 10$?

$V = 16000 - 1750 \times 10 = -1500 \Rightarrow$ This is impossible as $V > 0$. ①

Question continued

b i) $y = Ae^{kt}$ $\Rightarrow$ $V = Ae^{kt}$ 1 $\qquad e^0 = 1$

$t = 0,\ V = 16000$ $\Rightarrow$ $16000 = Ae^{k \times 0}$ $\Rightarrow$ $\underline{16000} = A$

$t = 4,\ V = 9000$ $\Rightarrow$ $9000 = 16000e^{4k}$ 1 $\qquad \frac{9000}{16000} = \frac{9}{16}$

$\Rightarrow$ $e^{4k} = 9/16$

$\Rightarrow$ $4k = \ln(9/16)$

$\Rightarrow$ $k = \frac{1}{4}\ln(9/16)$ 1

$\Rightarrow$ $V = \underline{16000e}^{1/4\ln(9/16)t}$ 1

b ii) $V = 16000e^{\frac{1}{4}\ln(9/16) \times 3}$

$V = 10392. \ldots$

$V = 10,\underline{400}$ barrels

(Total for Question is 7 marks)

8. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

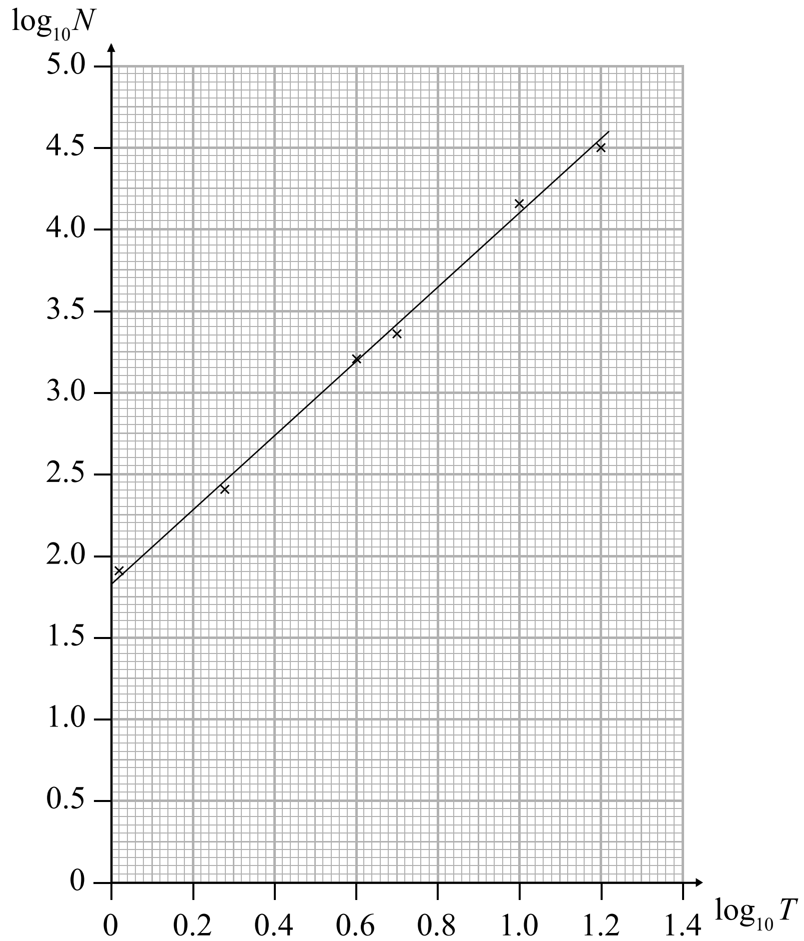


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

a) $N = aT^b$

$\log(ab) = \log(a) + \log(b)$
$\log(a^m) = m\log(a)$

$\log_{10}(N) = \log_{10}(aT^b)$
$= \log_{10}(a) + \log_{10}(T^b)$ (1)
$= \log_{10}(a) + b\log_{10}(T)$
$\Rightarrow \log_{10}(N) = m\log_{10}(T) + c$ where $m = b$ and $c = \log_{10}(a)$ (1)

b) $\log_{10} N = m\log_{10} T + c$

$N = aT^b$ $T = 3$

C : y-intercept $\Rightarrow C = 1.7$
$C = \log_{10}(a) \Rightarrow a = 10^{1.7} \approx 50.12$ $\Rightarrow a = 50.12$ (1)

$m = \frac{\Delta y}{\Delta x} = \frac{4.5 - 1.8}{1.2 - 0.1} = 2.46$ (1)

$\Rightarrow b = 2.46$
$\Rightarrow N = 50.12(3)^{2.46}$ (1) $= 747.7...$
$= 750$ microbes (1)

c) $N = 1000000 \Rightarrow \log_{10}(N) = \log_{10}(1000000) = 6$ (1)

$6 > 5.0$, which is outwith the data shown on the graph, which means that we can't extrapolate the data/graph, meaning that we can't assume that the model still holds. (1)

d) $N = aT^b$

let $T = 1 \Rightarrow N = a\,1^b$
$\Rightarrow N = a$

$\Rightarrow$ a is the number of microbes one day after the start of the experiment.

9. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

a) 6 months = 0.5 years
@ $t = 0.5$ - 1

$m = 25e^{-0.05 \times 0.5}$
$m = 24.4$g (3 s.f) - 1

b) $y = e^{kx}$
$\frac{dy}{dx} = ke^{kx}$

$\frac{dm}{dt} = -0.05 \times 25e^{-0.05t}$ - 1

$= -0.05m$

$\therefore k = -0.05$ - 1

10. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

$V = £100,000$

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

a)

i) in 2005, $t = 4$ $\Rightarrow$ $32000 = AP^4$
in 2012, $t = 11$ $\Rightarrow$ $50000 = AP^{11}$

$$\frac{\cancel{A}P^{11}}{\cancel{A}P^4} = \frac{50,000}{32,000}$$

$$p^7 = \frac{50,000}{32,000}$$

$$p = \sqrt[7]{\frac{50}{32}} \Rightarrow p = 1.0658$$

ii) $V = Ap^t$ $\Rightarrow$ $32,000 = A \times (1.0658)^4$

$$A = \frac{32000}{1.0658^4} = 24,799.73...$$

24,799.73... to 3 s.f. is 24,800

$\therefore A \approx 24,800$ as required.

Question continued

b)

$A = 24,800$ $p = 1.0658$ $V = Ap^t$

i) @ $t = 0$, $V = Ap^0 \Rightarrow V = A$

A is the value of the car on 1st january 2001.

ii) p is the factor by which the value increases/rises each year. The value rises by 6.6% each year.

Question continued

c) To find when $V = £100,000$

$V = 24,800 \times 1.0658^t$

$24,800 \times 1.0658^t = 100,000$

$1.0658^t = \dfrac{100,000}{24,800}$

$a^x = b, \; x = \log_a b$

$t = \log_{(1.0658)}\left(\dfrac{100,000}{24,800}\right)$

$t = 21.88$ years.

$\therefore$ The year the value of the car exceeds £100,000 is 2022.

(Total for Question is 10 marks)

11. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000 $\nearrow t=0$
- its value after one year is £16 000 $\rightarrow t=1$

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

a) $V = Ae^{kt}$ ↙ general exponential model ①

When car is new: $20000 = Ae^{k(0)}$ ①

$20000 = A(1) \quad \therefore A = 20000$

$V = 20000e^{kt}$

After one year: $16000 = 20000e^{k(1)}$

$16000 = 20000e^{k}$ ①

$e^{k} = \frac{4}{5}$

$\ln e^{k} = \ln\frac{4}{5}$

$k = \ln\frac{4}{5} = -0.223$

$V = 20000e^{-0.223t}$ ①

b) $V = 20\,000e^{-0.223(10)}$
$= £2150$ (1)

Actual value of car A after 10 years is £2000

£2150 ≈ £2000 ∴ Our model is reliable (1)

c) $V = Ae^{kt}$
'A' value will be the
same for car B since
car A and B have same
value when ∴ we have to ajust 'k' value

Make "-0.223" (the 'k' value) less negative (1)

12. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of $V \text{ km h}^{-1}$.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

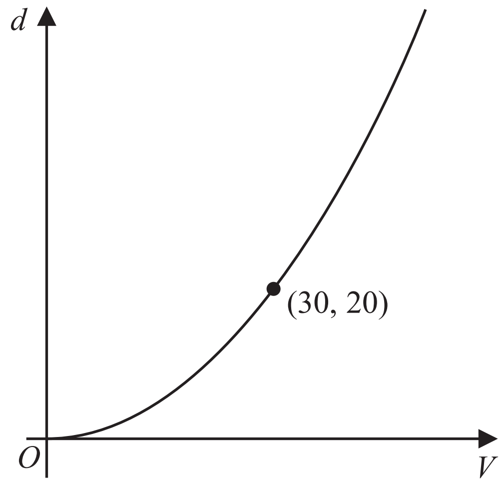


Figure 5

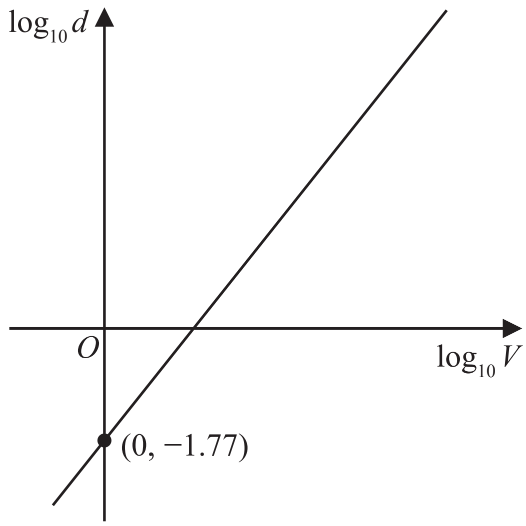


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

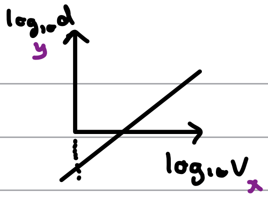
(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

a) $d = KV^n$, $K \approx 0.017$



Log laws:
$\log(ab) = \log(a) + \log(b)$
$\log(a^m) = m\log(a)$

$\log_{10}(d) = \log_{10}(KV^n)$
$= \log_{10}(K) + \log_{10}(V^n)$
$\log_{10}(d) = \log_{10}(K) + n\log_{10}V$

$K \approx 0.017$, $\log_{10}(0.017) = -1.7695...$ 1

$\Rightarrow \log_{10}(d) = \log_{10}(0.017) + n\log_{10}V$
$\Rightarrow \log_{10}(d) = n\log_{10}V - 1.77$ 2
$y \;\; = \;\; mx \; + c$

$\Rightarrow$ The linear nature and y-intercept in the equation matching that of figure 6 tells us that braking distance can be modelled by $\underline{d = KV^n}$. 3

b)
$d = 0.017 \cdot V^n$
For $(30, 20) \Rightarrow 20 = 0.017 \cdot 30^n$ 1
$\Rightarrow 30^n = \dfrac{20}{0.017}$

$\log(a^m) = m\log(a)$

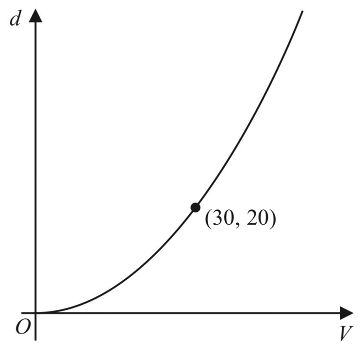


Figure 5

$\Rightarrow \log_{10}(30^n) = \log_{10}\left(\dfrac{20}{0.017}\right)$

$\Rightarrow n \cdot \log_{10}(30) = \log_{10}(20/0.017)$

$\Rightarrow n = \dfrac{\log_{10}\left(\frac{20}{0.017}\right)}{\log_{10}(30)} = 2.07876...$

$\Rightarrow n = 2.08$ 1 $\Rightarrow$ $\underline{d = 0.017 \cdot V^{2.08}}$ 1

c) $d = 0.017 \cdot V^{2.08}$ Distance: before + after breaks

$0.8s = \dfrac{0.8}{3600}$ hrs

Before brakes: $d_1 = \text{Speed} \times \text{time} = 60 \times \dfrac{0.8}{3600} = 0.0133...\text{ km}$
$\Rightarrow \underline{d_1 = 13.3\text{m}}$ 1

$d_2 = 0.017 \cdot 60^{2.08}$ 1
$d_2 = 84.9\text{m}$

$\Rightarrow$ Total distance $= 84.9 + 13.3 = 98.2\text{m}$ $\Rightarrow$ Sean stops in $\underline{\text{time}}$. 1

13. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation.*)

(2)

$n = Ae^{kt}$ 2 A and k are both positive constants.

We want an equation which is to do with exponential growth.

14. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

a) $\theta = A - Be^{-0.07t}$

$t = 0,\ \theta = 18 \Rightarrow 18 = A - Be^{-0.07 \cdot 0}$ $\quad e^{0} = 1$

$18 = A - B$ 1

$t = 10,\ \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$ 1

$A = \underline{B + 18}$ $\quad A = \underline{Be^{-0.7} + 44}$ $\Rightarrow B + 18 = Be^{-0.7} + 44$

$\Rightarrow B - Be^{-0.7} = 26$

$\Rightarrow B(\underline{1 - e^{-0.7}}) = 26$

$B = 51.6$ $\quad \Rightarrow B = \dfrac{26}{1 - e^{-0.7}} = 51.647\ldots \Rightarrow B = \underline{51.6}$ 1

$\Rightarrow A = 51.6 + 18 = \underline{69.6} = A$

$\Rightarrow \theta = \underline{69.6 - 51.6}e^{-0.07t}$ 1

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

b) $\theta = 69.6 - 51.6e^{-0.07t}$

The maximum temperature, according to the model, is 69.6 °C. 1
=> The model is not appropriate since 69.6 °C is much lower than 78 °C. 1

15. A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geqslant 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study. Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geqslant 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

(a) $t = 0$, $N = 1000$: $1000 = Ae^{0}$

$1000 = A$ 1

$\therefore N = 1000e^{kt}$

$t = 5$, $N = 2000$: $2000 = 1000e^{k(5)}$ 1

$2 = e^{5k}$

$\ln 2 = 5k$ 1

$k = \dfrac{\ln 2}{5}$

$\therefore N = 1000e^{\left(\frac{\ln 2}{5}\right)t}$ 1

(b) $\dfrac{dN}{dt} = \dfrac{\ln 2}{5} \times 1000e^{\left(\frac{\ln 2}{5}\right)t}$ 1 $= 200\ln 2 \times e^{\left(\frac{\ln 2}{5}\right)t}$

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Question continued

$t = 8$: $\frac{dN}{dt} = 200 \ln 2 \times e^{\left(\frac{\ln 2}{5}\right)(8)}$

$= 420.24\ldots$

$= 420$ (2sf) 1

(c) $500 e^{1.4 \times \left(\frac{\ln 2}{5}\right) \times T} = 1000 e^{\left(\frac{\ln 2}{5}\right) \times T}$ 1

$e^{0.28 \ln 2 \times T} = 2 e^{0.2 \ln 2 \times T}$

$\frac{e^{0.28 \ln 2 \times T}}{e^{0.2 \ln 2 \times T}} = 2$

$e^{0.08 \ln 2 \times T} = 2$

$0.08 \ln 2 \times T = \ln 2$ 1

$T = \frac{\ln 2}{0.08 \ln 2}$

$= 12.5$

$\therefore T = 12.5$ hours # 1